Can Horndeski gravity be recast in the Teleparallel framework?

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arXiv:1904.10791 in collaboration with Konstantinos F. Dialektopoulos and Jackson Levi Said

Outline

1. Horndeski gravity
   - From Galileons to Horndeski
   - Constraints on Horndeski from GWs
2. Teleparallel Horndeski gravity
   - Building the theory
   - GW in Teleparallel Horndeski
3. Conclusions

Outline

Galileons (in Minkowski background)

- Horndeski theory is the most general scalar tensor theory with one scalar field which leads to second order field eqs.
- The most straightforward way to obtain Horndeski theory is by considering Galileons; i.e. scalar fields that are invariant under the Galilean shift symmetry $\phi \to \phi + b_\mu x^\mu + c$.
- The most general Lagrangian that has the above property and gives second order field equations is

Lagrangian 2nd order Field eqs. for Minkowski

$$L = c_1\phi + c_2 X - c_3 X\Box\phi + c_4 X\left[(\Box\phi)^2 - \partial_\mu\partial_\nu\phi\partial^\mu\partial^\nu\phi\right] - c_5 X\left[(\Box\phi)^3 - 3(\Box\phi)\partial_\mu\partial_\nu\phi\partial^\mu\partial^\nu\phi + 2\partial_\mu\partial_\nu\phi\partial^\nu\partial^\lambda\phi\partial_\lambda\partial^\mu\phi\right],$$

where $X = -\frac{1}{2}\partial^\mu\phi\partial_\mu\phi$ is the kinetic term.

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Covariant Galileons

- To introduce gravity we promote $\eta_{\mu\nu} \to g_{\mu\nu}$ and $\partial_\mu \to \mathring{\nabla}_\mu$.
- Doing that, one needs to be careful since covariant derivatives do not commute. The naive covariantisation leads to higher derivatives in the field equations.
- Then, we need to add some correction terms, which leads the following Lagrangian

$$\mathring{L} = c_1\phi + c_2 X - c_3 X \mathring{\Box}\phi + \frac{c_4}{2}X^2\mathring{R} + c_4 X\left[(\mathring{\Box})^2 - \phi^{\mu\nu}\phi_{\mu\nu}\right] + c_5 X^2 \mathring{G}^{\mu\nu}\phi_{\mu\nu} - \frac{c_5}{3}X\left[(\mathring{\Box})^3 - 3\mathring{\Box}\phi^{\mu\nu}\phi_{\mu\nu} + 2\phi_{\mu\nu}\phi^{\nu\lambda}\phi^{\mu}_{\lambda}\right]. \quad (1)$$

and $\phi_{\mu\nu} = \mathring{\nabla}_\mu\mathring{\nabla}_\nu\phi$. Over-circles mean that it's computed with respect to Levi Civita connection.

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Horndeski gravity

- A generalized version of the above is the Horndeski one

$$\mathring{L}_2 = G_2(\phi, X)\,, \quad \mathring{L}_3 = -G_3(\phi, X)\mathring{\Box}\phi\,, \tag{2}$$

$$\mathring{L}_4 = G_4(\phi, X)\mathring{R} + G_{4,X}(\phi, X)\left[(\mathring{\Box}\phi)^2 - \mathring{\nabla}_\mu\mathring{\nabla}_\nu\phi\mathring{\nabla}^\mu\mathring{\nabla}^\nu\phi\right]\,, \tag{3}$$

$$\mathring{L}_5 = G_5(\phi, X)\mathring{G}_{\mu\nu}\mathring{\nabla}^\mu\mathring{\nabla}^\nu\phi - \frac{1}{6}G_{5,X}(\phi, X)\Big[(\mathring{\Box}\phi)^3 + 2\mathring{\nabla}_\nu\mathring{\nabla}_\mu\phi\mathring{\nabla}^\nu\mathring{\nabla}^\lambda\phi\mathring{\nabla}_\lambda\mathring{\nabla}^\mu\phi - 3\mathring{\Box}\phi\mathring{\nabla}_\mu\mathring{\nabla}_\nu\phi\mathring{\nabla}^\mu\mathring{\nabla}^\nu\phi\Big]\,, \tag{4}$$

where the total Lagrangian is $\mathring{L} = \sum_{i=2}^{5}\mathring{L}_i$.

Gravitational waves in standard Horndeski

- The speed of propagation of gravitational waves for Horndeski gravity in flat FLRW background is

Speed of GW in standard Horndeski

$$c_T^2 = \frac{G_4 - X(\ddot{\phi}G_{5,X} + G_{5,\phi})}{G_4 - 2XG_{4,X} - X(H\dot{\phi}G_{5,X} - G_{5,\phi})} \tag{5}$$

- According to GW observations *Prog.Theor.Phys. 126 (2011), 511-529*, it is required that $c_T = 1$ which is achieved only if $G_4(\phi, X) = G_4(\phi)$ and $G_5 = \mathrm{constant}$

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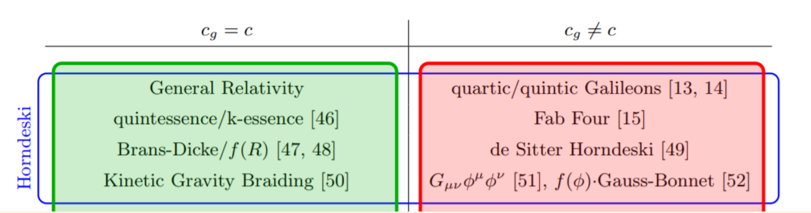

Figure: Summary of the viable (left) and non-viable (right) scalar-tensor theories after GW170817. PRL 119, 251304 (2017)

Outline

Torsion decomposition

The torsion tensor can be decomposed in its irreducible parts as

$$a_\mu = \frac{1}{6}\epsilon_{\mu\nu\sigma\rho}T^{\nu\sigma\rho}\,, \quad v_\mu = T^{\sigma}{}_{\sigma\mu}\,, \tag{6}$$

$$t_{\sigma\mu\nu} = \frac{1}{2}\left(T_{\sigma\mu\nu} + T_{\mu\sigma\nu}\right) + \frac{1}{6}\left(g_{\nu\sigma}v_\mu + g_{\nu\mu}v_\sigma\right) - \frac{1}{3}g_{\sigma\mu}v_\nu\,, \tag{7}$$

where $\epsilon_{\mu\nu\sigma\rho}$ is the totally anti-symmetric Levi-Civita symbol. From these we build the scalars

$$T_{\text{ax}} = a_\mu a^\mu\,, \quad T_{\text{vec}} = v_\mu v^\mu\,, \quad T_{\text{ten}} = t_{\sigma\mu\nu}t^{\sigma\mu\nu}\,, \tag{8}$$

and the torsion scalar is a linear combination

$$T = \frac{3}{2}T_{\text{ax}} + \frac{2}{3}T_{\text{ten}} - \frac{2}{3}T_{\text{vec}}\,. \tag{9}$$

Conditions for the theory

Condition 1

The resulting field equations must be at most second order in terms of derivatives of the tetrad fields (or equivalently second order in terms of metric tensor derivatives).

Condition 2

The scalar invariants should not be parity violating.

Condition 3

The field equations must be covariant under local Lorentz transformations.

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Contractions of the torsion tensor can at most be quadratic.

Any number of contractions of the irreducible parts of the torsion tensor will result in second order field equations. This means that an infinite number of terms can be formed in Teleparallel gravity that give rise to second order field equations. However, it is unclear how physical such higher order contributions will be.

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Covariantisation procedure

GR	Teleparallel
$\eta_{\mu\nu} \to g_{\mu\nu}$	$e^a{}_\mu \to h^a{}_\mu$
$\partial_\mu \to \mathring{\nabla}_\mu$	$\partial_\mu \to \mathcal{D}_\mu = \partial_\mu + h^c{}_\mu w^a{}_{bc} S^b_a$

Table: Covariantisation prescription

Following the same procedure as before, we start from

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Teleparallel Horndeski construction

- Since $\mathcal{D}_\mu$ coincides with $\mathring{\nabla}_\mu$ computed with the Levi-Civita connection, then the Teleparallel Lagrangians $\sum_{i=3}^{5}\mathcal{L}$ are identical to $\sum_{i=3}^{5}\mathring{L}_i$.
- However, when one is considering Teleparallel gravity, $\mathcal{L}_2$ would be different to $\mathring{L}_2$ since there are more scalars that one can construct which satisfies the conditions. Notation: $\mathcal{L}_2 = \mathring{L}_2 = G_2(\phi, X)$ and extra term $\mathcal{L}_{\rm Tele}$
- It might be that even so extra higher order derivatives term like $\mathcal{L}_6$ might appear. We would just concentrate on non higher-derivatives Lagrangian terms hereafter.

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Teleparallel Horndeski construction - Constructing $\mathcal{L}_{\rm Tele}$

- Taking quadratic contractions of the torsion tensor, the most general Lagrangian of Teleparallel gravity satisfying the conditions turns out to be $f(T_{\text{ax}}, T_{\text{vec}}, T_{\text{ten}})$ (without a scalar field).
- If one adds the scalar field, one can construct the following **7 extra independent scalars**:

Possible independent scalars

$$I_2 = v^{\mu}\phi_{;\mu}\,, \quad J_1 = a^{\mu}a^{\nu}\phi_{;\mu}\phi_{;\nu}\,, \quad J_3 = v_{\sigma}t^{\sigma\mu\nu}\phi_{;\mu}\phi_{;\nu}\,, \tag{11}$$

$$J_5 = t^{\sigma\mu\nu}t_{\sigma}{}^{\bar{\mu}}{}_{\nu}\phi_{;\mu}\phi_{;\bar{\mu}}\,, \quad J_6 = t^{\sigma\mu\nu}t_{\sigma}{}^{\bar{\mu}\bar{\nu}}\phi_{;\mu}\phi_{;\nu}\phi_{;\bar{\mu}}\phi_{;\bar{\nu}}\,, \tag{12}$$

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- Then, the extra term $\mathcal{L}_{\text{Tele}}$ related to Teleparallel gravity will be equal to

Extra term in $\mathcal{L}_2$ in Teleparallel Horndeski

$$\mathcal{L}_{\text{Tele}} = G_{\text{Tele}}(\phi, X, T, T_{\text{ax}}, T_{\text{vec}}, I_2, J_1, J_3, J_5, J_6, J_8, J_{10}) \,. \tag{14}$$

- It is equivalent to consider T_{ten} to T above.
- There are 10 independent scalars related to torsion and also contraction of it with the scalar field.

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Final Lagrangian Teleparallel Horndeski

The final Lagrangian reads

Lagrangian Teleparallel Horndeski

$$\mathcal{L} = \sum_{i=2}^{5} \mathcal{L}_i + \mathcal{L}_{\text{Tele}} = \text{Horndeski} + \mathcal{L}_{\text{Tele}}\,, \tag{15}$$

where

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$$\mathcal{L}_2 = G_2(\phi, X)\,, \quad \mathcal{L}_3 = G_3(\phi, X)\Box\phi\,,$$
$$\mathcal{L}_4 = G_4(\phi, X)\left(-T + B\right) + G_{4,X}(\phi, X)\left[\left(\Box\phi\right)^2 - \phi_{;\mu\nu}\phi^{;\mu\nu}\right]\,,$$
$$\mathcal{L}_5 = G_5(\phi, X)\mathcal{G}_{\mu\nu}\phi^{;\mu\nu} - \frac{1}{6}G_{5,X}(\phi, X)\Big[\left(\Box\phi\right)^3 + 2\phi_{;\mu}{}^{\nu}\phi_{;\nu}{}^{\alpha}\phi_{;\alpha}{}^{\mu} - 3\phi_{;\mu\nu}\phi^{\mu\nu}\left(\Box\phi\right)\Big]\,.$$

Some remarks about the Lagrangian

- In Horndeski gravity $f(R)$ does not appear since it is a 4th order theory. In Teleparallel Horndeski $f(T)$ appears in the Lagrangian.
- Teleparallel Horndeski has a richer structure since standard Horndeski is contained on it (setting $G_{\text{Tele}} = 0$).
- By redefining $\tilde{G}_{\text{Tele}} = G_{\text{Tele}} + TG_4(\phi, X)$ one gets theories with scalar field couplings with the boundary term alone.
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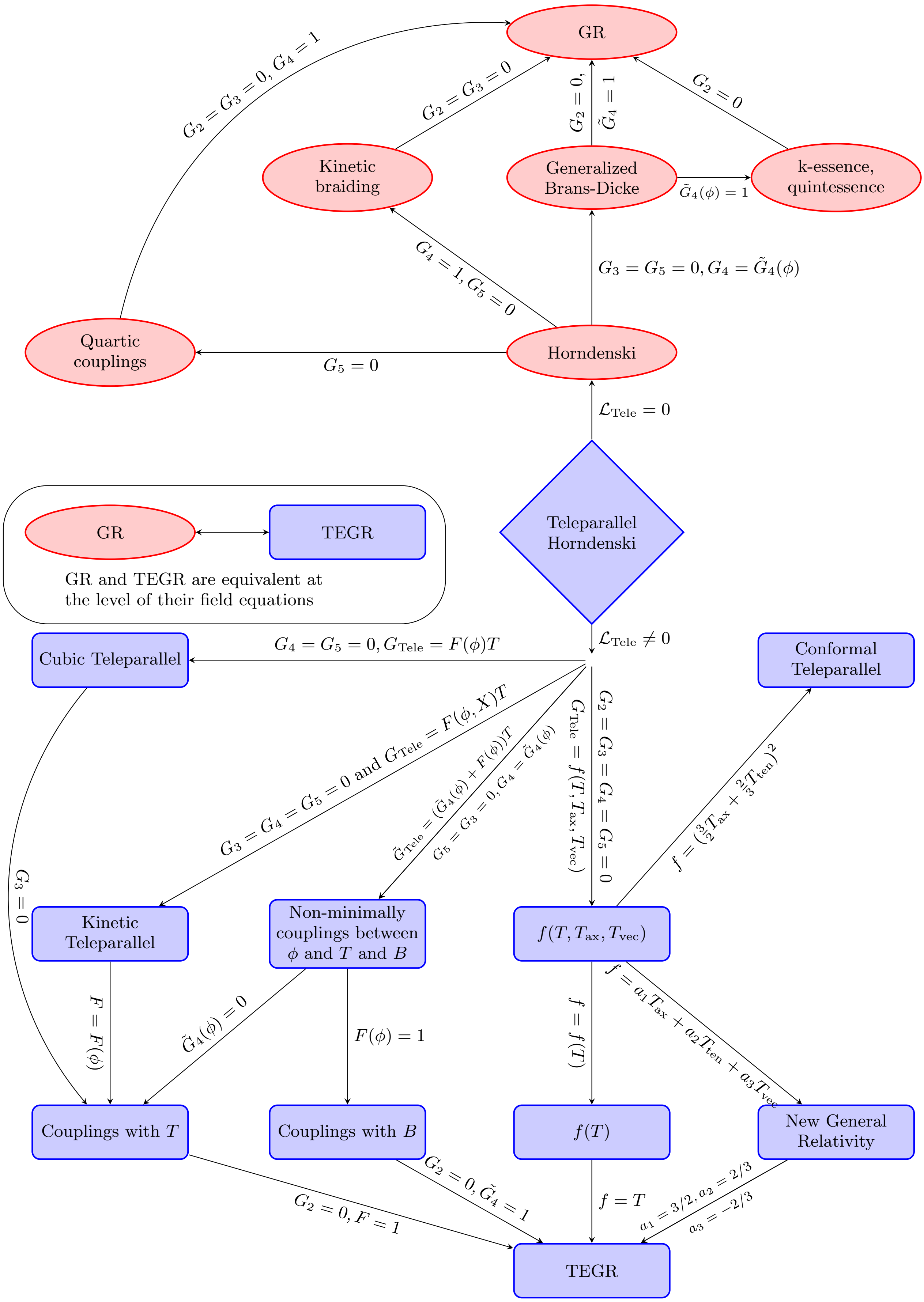

FIG. 1: Relationship between Teleparallel Horndenski and various theories.

Some remarks in flat FLRW cosmology

- In flat FLRW cosmology, $G_{\rm Tele} = G_{\rm Tele}(\phi, X, T, I_2)$ since the other scalars are identically zero or are not independent.
- We derived the FLRW equations as in the Horndeski case plus the corrections.
- Since $f(T)$ is a subclass of Teleparallel Horndeski, one can also conclude that Teleparallel Horndeski can explain both dark energy and inflation and also can alleviate the H_0 tension*.
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Gravitational waves in Teleparallel Horndeski in flat FLRW background

- By considering tensorial perturbations only and after some cumbersome calculations, one gets the following wave equation

$$\ddot{h}_{ij} + (3 + \alpha_{\mathsf{M}})\, H\dot{h}_{ij} - (1 + \alpha_{\mathsf{T}})\, \frac{k^2}{a^2} h_{ij} = 0, \tag{16}$$

where $\alpha_T = c_T^2 - 1$ and the speed of GW being equal to

Speed of GW in Teleparallel Horndeski

$$c_T^2 = \frac{G_4 - X(\ddot{\phi}G_{5,X} + G_{5,\phi}) - G_{\mathrm{Tele,T}}}{G_4 - 2XG_{4,X} - X(H\dot{\phi}G_{5,X} - G_{5,\phi}) + 2XG_{\mathrm{Tele,J_8}} + \frac{1}{2}XG_{\mathrm{Tele,J_5}} - G_{\mathrm{Tele,T}}}. \tag{17}$$

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Reviving Horndeski using Teleparallel gravity

- As we said before, for $G_{\rm Tele}=0$ (standard case), one gets that to achieve a theory consistent with the GW observations $c_T=1$, one requires $G_5(\phi,X)=\text{constant}$ and $G_4(\phi,X)=G_4(\phi)$. Hence, Horndeski gravity is highly constraint.
- If one has Teleparallel Horndeski, c_T^2 is corrected and then when does no need those conditions. Indeed, $G_5=G_5(\phi)$ and $G_4=G_4(\phi,X)$ still respect this condition.
- The theory which respects this condition is

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Outline

Conclusions

- Horndeski is the most general second order field equations with one scalar field. This theory was highly constraint after GW170817.
- We formulate an analogue version of it in the Teleparallel framework. Since torsion has first derivatives of tetrads, there are more terms that respects the second order condition.
- The Lagrangians $\mathring{L}_3, \mathring{L}_4$ and $\mathring{L}_5$ are the same in Horndeski and for Teleparallel Horndeski but $\mathring{L}_2$ differs by a term $\mathcal{L}_{\text{Tele}} = G_{\text{Tele}}(\phi, X, T, T_{\text{ax}}, T_{\text{vec}}, I_2, J_1, J_3, J_5, J_6, J_8, J_{10})$.
- After analysing tensorial perturbations for GW, we found that for Teleparallel Horndeski, one can restore the terms $G_5 = G_5(\phi)$ and $G_4 = G_4(\phi, X)$ giving a consistent theory with $c_T = 1$ ($c_g = c$).

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